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BUSINESS ADMINISTRATION

QUANTITATIVE METHODS

JORGE SERRANO

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1. INTRODUCTION

The objective of the lecture is learning tools and techniques for comparing and deciding between investment alternatives.

When deciding between investment alternatives we must project into the future the most probable cash flow forecast for each alternative in order to asses which is the most profitable choice. In this comparison it will be needed to deal with the problem of trying to compare the value of different cash flows located in different moments.

As we all know by intuition, the same amount of money has more value today than far into the future. But, how much more?. To answer this kind of questions it will be needed firstly to learn some basic principles of financial mathematics.

After learning some financial concepts, we will show different methods available to compare and decide between capital projects.

2. BASIC PRINCIPLES OF FINANCIAL MATHEMATICS

2.1 Interest:

What does it mean?

It is the charge for the privilege of borrowing money, typically expressed as an annual percentage rate. Lenders make money from interest, borrowers pay it.

Simple Interest

It is a quick method of calculating the interest charge on a loan. Simple interest is determined by multiplying the interest rate by the principal by the number of periods.

$I = P \times i \times N$

Where:
I is the interest charged
P is the loan amount
i is the interest rate
N is the duration of the loan, using number of periods

Simple interest is called simple because it ignores the effects of compounding. The interest charge is always based on the original principal, so interest on interest is not included. This method may be used to find the interest charge for short-term loans, where ignoring compounding is less of an issue.

Compound interest

Compound interest is the interest that accrues on the initial principal and the accumulated interest of a principal deposit, loan or debt. Compounding allows a principal amount to grow at a faster rate than simple interest, which is calculated as a percentage of the principal amount.

The more frequently interest is added to the principal, the faster the principal grows and the higher the compound interest will be. The frequency at which the interest is compounded is established at the initial stages of securing the loan. Generally, interest tends to be calculated on an annual basis although other terms may be established at the time of the loan.

2.2 Time Value

Imagine you have won a cash prize and you have two payment options:

A. Receive 1.000 € now

B. Receive 1.000 € in three years

Which option would you choose? If you're like most people, you would choose to receive the 1.000 € now. After all, three years is a long time to wait. Why would any rational person defer payment into the future when he or she could have the same amount of money now?

For most of us, taking the money in the present is just plain instinctive. So at the most basic level, the time value of money demonstrates that, all things being equal, it is better to have money now rather than later.

But why is this? A 1.000 € bill has the same value as a 1.000 € bill one year from now, doesn't it? Actually, although the bill is the same, you can do much more with the money if you have it now: over time you can earn more interest on your money.

Back to our example: By receiving 1.000 € today, you are able to increase the future value of your money by investing and gaining interest over a period of time. For option B, you don't have time on your side, and the payment received in three years would be your future value.

If you are choosing option A, your future value will be 1.000 € plus any interest acquired over the three years. The future value for option B, on the other hand, would only be 1.000 €. But stay tuned to find out how to calculate exactly how much more option A is worth, compared to option B.

Future Value Basics

If you choose option A and invest the total amount at a simple annual rate of 4.5%, the future value of your investment at the end of the first year is 1.045 € which of course is calculated by multiplying the principal amount of 1.000 € by the interest rate of 4.5% and then adding the interest gained to the principal amount.

You can also calculate the total amount of a one-year investment with a simple manipulation of the above equation:

- Original equation: (1.000 x 0.045) + 1.000 = 1.045
- Manipulation: 1.000 x [(1 x 0.045) + 1] = 1.045
- Final equation: 1.000 x (0.045 + 1) = 1.045

If the 1.045 € left in your investment account at the end of the first year is left untouched and you invested it at 4.5% for another year, how much would you have? To calculate this, you would take the 1.045 € and multiply it again by 1.045 (0.045 +1). At the end of two years, you would have 1.092 €:

The above calculation, then, is equivalent to the following equation:

$$\text{Future Value} = 1.000 \times (1+0.045) \times (1+0.045)$$

Think back to math class in junior high, where you learned the rule of exponents, which says that the multiplication of like terms is equivalent to adding their exponents. In the above equation, the two like terms are (1+0.045), and the exponent on each is equal to 1. Therefore, the equation can be represented as the following:

Future Value = 1.000 x $(1+0.045)^2$ = 1.092

We can see that the exponent is equal to the number of years for which the money is earning interest in an investment. So, the equation for calculating the three-year future value of the investment would look like this:

Future Value = 1.000 x $(1+0.045)^3$ = 1.141

This calculation shows us that we don't need to calculate the future value after the first year, then the second year, then the third year, and so on. If you know how many years you would like to hold a present amount of money in an investment, the future value of that amount is calculated by the following equation:

Future Value = Present Value x (1 + interest rate) ^ number of periods

Present Value Basics

If you received 1.000 € today, the present value would of course be 1.000 € because present value is what your investment gives you now if you were to spend it today. If 1.000 € were to be received in a year, the present value of the amount would not be 1.000 € because you do not have it in your hand now, in the present. To find the present value of the 1.000 € you will receive in the future, you need to pretend that the 1.000 € is the total future value of an amount that you invested today. In other words, to find the present value of the future 1.000 € we need to find out how much we would have to invest today in order to receive that 1.000 € in the future.

To calculate present value, or the amount that we would have to invest today, you must subtract the (hypothetical) accumulated interest from the 1.000 €. To achieve this, we can discount the future payment amount by the interest rate for the period. The above future value equation can be rewritten as follows:

Future Value = Present Value x (1 + interest rate) ^ number of periods

Present Value = Future Value / (1 + interest rate) ^ number of periods

Let's walk backwards from the 1.000 € offered in option B. Remember, the 1.000 € to be received in three years is really the same as the future value of an investment. So, if we want to calculate today's present value of the 1.000 € expected from a three-year investment earning 4.5%, the needed equation would be as follows:

Present Value = 1.000 / $(1+0.045)^3$ = 876

So the present value of a future payment of 1.000 € is worth 876 € today if interest rates are 4.5% per year. In other words, choosing option B is like taking 876 € now and then investing it for three years. The equations above illustrate that option A is better not only because it offers you money right now but because it offers you 124 € more in cash! Furthermore, if you invest the 1.000 € that you receive from option A, your choice gives you a future value that is 141 € (1.141- 1.000) greater than the future value of option B.

Conclusion
These calculations demonstrate that time literally is money - the value of the money you have now is not the same as it will be in the future and vice versa. So, it is important to know how to calculate the time value of money so that you can distinguish between the worth of investments that offer you returns at different times.

2.3 Compounding and discounting

Compounding

It is the ability of an asset to generate earnings, which are then reinvested in order to generate their own earnings. In other words, compounding refers to generating earnings from previous earnings.

Suppose you invest 10.000 € by buying shares. The first year, the shares rise 20%. Your investment is now worth 12.000 €. Based on good performance, you hold the stock. In Year 2, the shares appreciate another 20%. Therefore, your 12.000 € grows to 14.400 €. Rather than your shares appreciating an additional 2.000 € (20%) like they did in the first year, they appreciate an additional 400 €, because the 2.000 € you gained in the first year grew by 20% too. If you extrapolate the process out, the numbers can start to get very big as your previous earnings start to provide returns. In fact, 10.000 € invested at 20% annually for 25 years would grow to nearly 1.000.000 € (and that's without adding any money to the investment)!

The power of compounding was said to be deemed the eighth wonder of the world - or so the story goes - by Albert Einstein.

Compound Annual Growth Rate (CAGR)

It is the year-over-year growth rate of an investment over a specified period of time.

The compound annual growth rate is calculated by taking the nth root of the total percentage growth rate, where n is the number of years in the period being considered.

This can be written as follows:

$$CAGR = \left(\frac{\text{Ending Value}}{\text{Beginning Value}}\right)^{\left(\frac{1}{\#\text{ of years}}\right)} - 1$$

CAGR isn't the actual return in reality. It's an imaginary number that describes the rate at which an investment would have grown if it grew at a steady rate. You can think of CAGR as a way to smooth out the returns.

CAGR represents the smoothed annualized gain you earned over your investment time horizon.

Equivalent Annual Rate (EAR)

Since an interest rate may be defined using different compounding terms (daily, monthly, annually, or other), the annual "cost" of interest (nominal interest rate) between two different loans may not be comparable. The Equivalent Annual Rate rate is used to make such loans more comparable by converting any loan into the equivalent annual rate.

The EAR is calculated in the following way, where, i the nominal interest rate, and n the number of compounding periods per year:

EAR = (1 + i/n)^n - 1

For example, a savings account with a nominal interest rate of 10% that pays interest quarterly would have an equivalent annual rate of 10.38%. Investors should be aware that the equivalent annual rate will typically be higher than the actual annual rate calculated without compounding.

Discount Rate

It is the interest rate used in determining the present value of future cash flows.

For example, let's say you expect 1.000 € in one year's time. To determine the present value of this 1000 € (what it is worth to you today) you would need to discount it by a particular rate of interest (often the risk-free rate but not always). Assuming a discount rate of 10%, the 1.000 € in a year's time would be the equivalent of 909,09 € to you today (1000/[1.00 + 0.10]).

Discounting

It is the process of finding the present value of an amount of cash at some future date. The discounted value of a cash flow is determined by reducing its value by the appropriate discount rate for each unit of time between the time when the cashflow is to be valued to the time of the cash flow. Most often the discount rate is expressed as an annual rate.

To calculate the present value of a single cash flow, it is divided by one plus the interest rate for each period of time that will pass. This is expressed mathematically as raising the divisor to the power of the number of units of time.

As an example, suppose an individual wants to find the present value of 100 € that will be received in five years time. There is a question of how much is it worth presently, and what amount of money, if one lets it grow at the discount rate, would equal 100 € in five years.

Let one assume a 12% per year interest rate.

PV = 100 dollars divided by 1 plus 12% (0.12) to the power 5

The present value is about 56,74 €.

2.4 Cash flow:

The Essentials Of Cash Flow

If a company reports earnings of 1 billion €, does this mean it has this amount of cash in the bank? Not necessarily. Financial statements are based on accrual accounting, which takes into account non-cash items. It does this in an effort to best reflect the financial health of a company. However, accrual accounting may create accounting noise, which sometimes needs to be tuned out so that it's clear how much actual cash a company is generating. The statement of cash flow provides this information, and here we look at what cash flow is and how to read the cash flow statement.

What Is Cash Flow?

Business is all about trade, the exchange of value between two or more parties, and cash is the asset needed for participation in the economic system. For this reason - while some industries are more cash intensive than others - no business can survive in the long run without generating positive cash flow per share for its shareholders. To have a positive cash flow, the company's long-term cash inflows need to exceed its long-term cash outflows.

An outflow of cash occurs when a company transfers funds to another party (either physically or electronically). Such a transfer could be made to pay for employees, suppliers and creditors, or to purchase long-term assets and investments, or even pay for legal expenses and lawsuit settlements. It is important to note that legal transfers of value through debt - a purchase made on credit - is not recorded as a cash outflow until the money actually leaves the company's hands.

A cash inflow is of course the exact opposite; it is any transfer of money that comes into the company's possession. Typically, the majority of a company's cash inflows are from customers, lenders (such as banks or bondholders) and investors who purchase company equity from the company. Occasionally cash flows come from sources like legal settlements or the sale of company real estate or equipment.

Cash Flow vs Income

It is important to note the distinction between being profitable and having positive cash flow transactions: just because a company is bringing in cash does not mean it is making a profit (and vice versa).

For example, say a manufacturing company is experiencing low product demand and therefore decides to sell off half its factory equipment at liquidation prices. It will receive cash from the buyer for the used equipment, but the manufacturing company is definitely losing money on the sale: it would prefer to use the equipment to manufacture products and earn an operating profit. But since it cannot, the next best option is to sell off the equipment at prices much lower than the company paid for it. In the year that it sold the equipment, the company would end up with a strong positive cash flow, but its current and future earnings potential would be fairly bleak. Because cash flow can be positive while profitability is negative, investors should analyze income statements as well as cash flow statements, not just one or the other.

3. INVESTMENT ANALYSIS

Payback Period

Various techniques have been developed for application to individual projects. The simplest individual technique is the **payback period**—the time required for total cash inflows to equal total cash outflows. Projects are ranked according to payback period, and accepted if the payback period is below some maximum length. While simple to compute, there is no generally accepted method to set the maximum payback period, the time value of money is not considered, and cash flows past the payback period are ignored.

Discounted cash flow techniques:

Discounted cash flow (DCF) techniques are preferable because they consider the time value of money, are based on actual cash flows rather than accounting profits, and have a definite standard. These techniques compare the rate of return from a project to the rate of return available on other investments of similar risk—a comparison of marginal return to marginal cost. The two widely used DCF techniques are based on the concept of present value. The present value of a series of cash flows is the amount that, if invested at the required rate of return for the project, will re-create the expected cash flows from the project.

Net present Value:

The net present value (NPV) is computed as the present value of the project cash flows minus the cost of the project. If NPV is negative, the present value of the cash flows from the project is less than the cost of the project—i.e., it would be cheaper to generate the cash flows by investing at the required rate than by undertaking the project. Since the cash flows could be created more cheaply by investing at the required rate, the project rate of return is below the required rate, and rejection is indicated. Alternately stated, rejecting the project and investing the cost of the project elsewhere would create larger cash flows than accepting the project. Where NPV is positive, it is cheaper to generate the cash flows by undertaking the project than by investing at the required rate of return—i.e., the project rate of return is greater than the required return, and acceptance is indicated. Alternately stated, investing the project creates larger cash flows than investing elsewhere. NPV is sometimes described as the change in the value of the firm if the project is accepted.

Internal Rate of Return:

The internal rate of return (IRR) is the rate of return that would be required to exactly re-create the cash flows from an investment equal to the cost of the project. The IRR is the rate of return provided by the project if accepted. If the IRR is greater than the required rate of return, acceptance is indicated. If the IRR is below the required rate of return, rejection is indicated.

While the IRR technique appears simpler to understand and apply, it can be misleading if there is a limit to the number of projects that can be accepted or if projects are mutually exclusive, so that a ranking technique is necessary. While IRR and NPV will always give the same accept/reject decision, the ranking of projects on IRR may differ from the ranking on NPV. This difference arises because of differences in the implicit assumptions about the rate of return on cash flows from the project. The NPV technique implicitly assumes reinvestment of cash flows at the required rate of return, while the IRR implicitly assumes reinvestment of cash flows at the IRR. The NPV criterion is considered superior because it is likely that the actual reinvestment rate will be close to the required rate. The IRR, on the other hand, may depart widely from the actual reinvestment rate. The reinvestment rate is also important when considering projects of different lives. It is possible that a long project of lower return may be preferable to a short project of high return if the cash flows from the short project will be reinvested at a low rate. Where information about the actual reinvestment rate is obtainable, however, both NPV and IRR can be modified to reflect this rate. This is accomplished by compounding all cash flows forward until the end of the project.

Limitations of quantitative techniques

An accept or reject indication on the above criteria does not mean that a project should be automatically accepted or rejected. Many other factors need to be considered. First, the criteria are based on estimated cash flows and an estimated required rate. The estimates are themselves subject to uncertainty and this may lead to an increase or safety factor in the "hurdle." Second, as noted, the estimates proceed on an individual project basis, and there may be an interaction between projects. Third, and perhaps most important, the criteria consider only cash flows, and some factors cannot be reduced to a monetary basis. It must be remembered that a capital budget is in reality the strategy chosen to reach the goals of the firm. The indications of the quantitative economic analysis are only a part of the strategic planning process and are subsidiary to overall strategic considerations. Unless a project is compatible with the goals of the firm, it will not be accepted. Conversely, if a project has non monetary benefits or interaction with other projects, it may be accepted despite a negative indication. Again, the capital budget is a planning document. The greatest contribution of the application of the capital budgeting techniques is not the indicated decision, but the heuristic benefits of greater understanding.

Finally, ethical standards are a vital part of the strategic considerations. An otherwise acceptable project may be unacceptable on ethical grounds. The social impact of projects has become increasingly important. It is necessary to consider the externalities—the effects of the project that are not felt by the firm. Externalities include such items as environmental impact and required increase in infrastructure.

Sources:

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